

Small Ball Probabilities for the Poisson Process and Applications

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Abstract

For centered jump Lévy processes we study the small ball probabilities. Namely, for a process X which is a centered pure jump Lévy process and comes close to the Wiener process (when intensity of jumps r tends to ∞) we are interested in the probability $\Pr\{\sup_{t \in [0,1]} |X(t) - f(t)| \leq r\}$ for admissible shift functions f as $r \rightarrow 0$. For this probability we obtained an asymptotic estimate which coincides with the known Wiener-type estimate, when intensity r increases “fast”. This asymptotic estimate is different, when intensity changes “slower”.

We discuss also various applications of small ball probability estimates. For example, we apply these estimates to prove the functional Strassen law of the iterated logarithm for the empirical processes, to estimate the quantization error in approximation theory, coding theory. We discuss also their applications for reliability theory.